

ABSTRACTS

Complexity and Nonextensivity:

New Trends in Statistical Mechanics (CN-Kyoto)

14-18 March, 2005

**The Yukawa Institute for Theoretical Physics,
Kyoto University, Kyoto, Japan**

Oral Presentations

Scale-invariant occupation of phase space and additive entropy S_q

Constantino Tsallis

Santa Fe Institute, 1399 Hyde Park Road, Santa Fe, NM, 87501, USA

and

Centro Brasileiro de Pesquisas Físicas, Xavier Sigaud 150,

Rio de Janeiro 22290-180, Brazil

Phase space is constructed through N subsystems that might be (probabilistically) independent or correlated. If they are independent, Boltzmann-Gibbs entropy S_{BG} $\equiv -k \sum_i p_i \ln p_i$ is strictly additive in the sense that the entropy $S_{\text{BG}}(N)$ associated with N subsystems equals $N S_{\text{BG}}(1)$. If they are correlated, two possibilities might occur. Either the correlations between distant parts of phase space generically tend to disappear at the thermodynamic limit ($N \rightarrow \infty$), or they remain finite at all scales. In the first case, S_{BG} is asymptotically additive, i.e., extensive ($\lim_{N \rightarrow \infty} S_{\text{BG}}(N)/N < \infty$). For some classes of the second case (e.g., standard critical phenomena), we can still advantageously use S_{BG} , verifying however the emergence, in the theory, of a variety of singular functions. There are however important classes of the second case (typically many of the so-called complex systems), for which we definitively need to focus on a different functional for the entropy. We exhibit two paradigmatic systems (one discrete and one continuous) for which the entropy $S_q \equiv (1 - \sum_i p_i^q)/(q-1)$ is strictly additive, whereas S_{BG} is neither strictly nor

asymptotically so. Both of our examples include physically suggestive scale-invariant correlations.

Bibliography: C. Tsallis, M. Gell-Mann and Y. Sato, preprint (2005).

**Quasistationary states and arrested dynamics near critical attractors
in unimodal maps**

Alberto Robledo

*Instituto de Fisica, Universidad Nacional Autonoma de Mexico,
Apartado Postal 20-364, Mexico 01000 DF, Mexico*

A key feature of the nonextensive statistics is its crossover to the Boltzmann-Gibbs (BG) statistics. In this talk we illustrate the mechanisms for this crossover at critical attractors in one-dimensional maps. First, we consider the tangent bifurcation and explain how this aspect is related to the feedback feature from chaotic regions into the neighborhood of the tangency and describe the implications to the related slow dynamics at thermal critical states. Secondly, we consider the edge of chaos via period doubling bifurcations and discuss two different ways to bring about the crossover. One corresponds to a shift of the map into a 2^n -band chaotic attractor and the other to a perturbation of the Feigenbaum attractor with additive noise. We show that in the latter case we obtain all the properties of glassy dynamics close to vitrification in molecular glass formers. In all cases we find that the critical dynamics is based on a set of Mori's q -phase transitions and find that the value of q at each transition corresponds to the same special value for the entropic index q , such that the resultant sets of q -Lyapunov coefficients are equal to the Tsallis rates of entropy change.

**Nonextensive thermodynamics of a cluster consisting
of M Hubbard dimers ($M=1,2,3$ and ∞)**

Hideo Hasegawa

*Dept. of Physics, Tokyo Gakugei University, 4-1-1 Nukui-kita machi, Koganei,
Tokyo 184-8501, Japan*

The thermodynamical property of a small cluster including M Hubbard dimers, each of which is described by the two-site Hubbard model, has been discussed within the nonextensive statistics (NES). The temperature dependence of the energy, entropy, specific heat and susceptibility has been calculated for $M = 1, 2, 3$ and ∞ (corresponding to $q=1$) with the use of the relation between the entropic index q and the cluster size N given by $q=1+2/N$ ($N = 2 \times M$ for dimers), which was previously derived by several methods. We have pointed out that although the joint probability of a system consisting of independent subsystems is conventionally expressed {it a priori} as the product of the probabilities of constituent elements, it is {it not} necessarily the case in the NES, which is supported by our model calculations.

Stochastic non-ergodicity

Eli Barkai

*Physics Department, Bar-Ilan University,
52900 Ramat-Gan, Israel*

A non-ergodic statistical mechanical framework is briefly presented. The theory

reduces to Boltzmann--Gibbs statistical mechanics in the ergodic phase. In the non-ergodic phase the theory describes ergodicity breaking in models and systems with power law sojourn times, for example the continuous time random walk, and blinking quantum dots. In this talk we will consider the non-ergodicity properties of chaotic maps. Connections between ergodicity breaking, anomalous diffusion, q -exponential sensitivity on initial conditions, are discussed.

Nonextensivity at the chaos threshold of the z -logistic map: Connection between the relaxation and the average sensitivity entropic indices

Ugur Tirnakli

Department of Physics, Faculty of Science, Ege University, 35100 Izmir, Turkey

By analysing the ensemble averages of the sensitivity to initial conditions and the entropy increase rates of the z -logistic map family at the chaos threshold, we numerically verify that the Pesin theorem is valid not only for cycle-2 but also for all other cycles that appear at periodic windows in chaotic region. We also show, for the first time, that the two families of entropic index q , namely, the one that comes from the properties of sensitivity to initial conditions (denoted by $q_{\text{sen}}^{\text{av}}$) and the one that comes from relaxation dynamics (denoted by q_{rel}) are related to each other through a simple scaling form. Moreover, we find similar scaling relations among the $q_{\text{sen}}^{\text{av}}$ values of each cycle.

Refined formalism of the maximum entropy principle in Tsallis statistics

Hiroki Suyari

*Department of Information and Image Sciences, Faculty of Engineering,
Chiba University, 1-33 Yayoi-cho, Inage-ku, Chiba-shi, Chiba 263-8522, Japan*

The maximum entropy principle in Tsallis statistics is reformulated in the mathematical framework of the q -product, which results in much simpler and natural distribution than usual one. As one of the applications of the present formalism, we theoretically derive the physical temperature which coincides with that obtained by Abe et al..

References:

- [1] C. Tsallis, R.S. Mendes, A.R. Plastino, The role of constraints within generalized nonextensive statistics, *Physica A* 261, 534-554 (1998).
- [2] H. Suyari, M. Tsukada, Law of error in Tsallis statistics, *IEEE Trans. Inform. Theory.*, vol.51, pp.753-757 (2005).
- [3] H. Suyari, Mathematical structure derived from the q -multinomial coefficient in Tsallis statistics, cond-mat/0401546.
- [4] S. Abe, S. Martinez, F. Pennini, A. Plastino, Nonextensive thermodynamic relations, *Phys. Lett. A* 281, pp.126-130 (2001).

**Quantum Fisher information and q -deformed relative entropies:
additivity vs nonadditivity**

Hiroshi Hasegawa

*Institute of Quantum Science, College of Science and Technology, Nihon University,
1-8-14 Kanda-Surugadai, Chiyoda-ku, Tokyo 101-8308, Japan*

We discuss precise relationship between quantum Fisher information metrics and quantum q -deformed relative entropies with non-additive nature. A remarkable result we have obtained is that, in spite of general nonadditivity of the latter quantities, the quantum Fisher metrics derived there-from are additive, which is confirmed in several examples. We investigate its reason from quantum-information theoretical viewpoint to establish a general proof.

**Simulation of stationary states of the two dimensional electron plasma trapped
in magnetic field**

Ryo Kawahara

*Department of Physics, Kyushu University, 6-10-1, Hakozaki, Higashi-ku,
Fukuoka-city, 812-8581, Japan*

Pure electron plasma can be trapped in a vessel by magnetic field, which is called Malmberg trap. Its macroscopic dynamics is well approximated by the Euler equation of the two dimensional (2-d) incompressible inviscid fluid with vortices of the same sign; this suggests the close relationship between the statistics of freely decaying 2-d turbulence and that of the 2-d electron plasma.

Experiments revealed that there are several kinds of stationary states for this system, such as vortex crystals, single peak distributions, etc., and the statistical description of them has been developed.

Since the electrons have the long range interactions of the Coulomb forces, the Boltzmann-Gibbs statistics may not be applied at the thermodynamical limit, therefore, several alternative theories have been developed, such as maximization of fluid entropy or minimization of enstrophy. The latter is known to be identical to the maximization of Tsallis entropy for $q = 1/2$.

We have done computer simulations of the 2-d point vortex model and compared the results to those statistical theories. We have found that the final states depend on the initial states, and some of them are close to the minimum enstrophy state, but most of them are not.

Functional networks: Structure of noise and flow

Bosiljka Tadic

*Theoretical Physics Department, Jozef Stefan Institute,
1001 Ljubljana, Slovenia*

Structured networks, in particular scale-free topologies, may influence in different ways the dynamic processes taking part on them. In general, more efficient processes occur on networks with higher structural complexity.

Considering functional networks, the relevant topological properties may differ considerably from pure geometry of links.

In this lecture we discuss how the differences between topology and function emerge within a numerical model of information transport on networks. In particular, we determine the dynamic analogue of the topological betweenness and betweenness-centrality (betweenness of nodes and links). In real functional networks these properties are known as (multichannel) noise and network flow. We demonstrate how properties of noise and flow are related to the global transport efficiency and to the network structure. Furthermore, we discuss potential use of the noise and flow analysis for the network reconstruction and optimization of the transport processes on networks.

Complex Networks in Psychological Models

Roseli S. Wedemann

*Departamento de Ciencia da Computacion, Instituto de Matematica e Estatistica,
Universidade do Estado do Rio de Janeiro, Rua St. Francisco Xavier, 524,
20550-013, Rio de Janeiro, RJ, Brazil*

Since little is still known about fundamental brain mechanisms associated to thought, its different manifestations are usually classified in an oversimplified way into normal and abnormal, like creative thinking or delusional and disorganized thought. Considering dopaminergic signal-to-noise neuronal modulation in the central nervous system, and the existence of semantic maps in the human brain, we propose a schematic self-organizing neural network model, to explain development of cortical map structure and dynamics of memory access, and unify different mental processes into a single neurocomputational substrate.

Based on neural network models, neurotic behavior, basically the compulsion to repeat neurotic symptoms as described by Freud, may be understood as an associative memory process in the brain, and the linguistic, symbolic, associative process involved in psychoanalytic working-through can be mapped onto a corresponding process of reconfiguration of the neural network. The model is illustrated through computer simulations, where we varied dopaminergic modulation and observed the self-organizing emergent patterns at the resulting semantic map, interpreting them as different manifestations of thought. We relate sensitivity to noise and adaptive capabilities of our model, with the sensitivity of cortical map modulation to the catecholamines (norepinephrine and dopamine). The signal-to-noise ratio regulated by these substances influences thought associativity, suggesting a continuous potentiality for the creation of different mental structures, from psychotic through to normal and neurotic behavior, and creativity.

**On a nongrowing small-world and scale-free network model
with geographical consideration**

Naoki Masuda*, Hiroyoshi Miwa**, Norio Konno***

**Laboratory for Mathematical Neuroscience, RIKEN BSI, 2-1 Hirosawa, Wako,
Saitama 351-0198, Japan*

***Department of Informatics, School of Science and Technology,
Kwansei Gakuin University, 2-1, Gakuen, Sanda, Hyogo, 669-1337 Japan*

****Faculty of Engineering, Yokohama National University, 79-5,
Tokiwadai, Hodogaya, Yokohama, 240-8501 Japan*

Many real networks are complex and equipped with power-law vertex degree distributions, short diameter, and clustering. Barabasi and Albert proposed a growing network model that accommodates these properties. However, real networks are not necessarily growing. We present a network model created by thresholding the summed vertex weights and also extend the model to geographical cases. More specifically, each vertex is endowed with a random weight, and edges form when a pair of vertices are spatially close and/or have large summed weights. Scale-free and small-world networks are produced for a wide class of weight distributions. The geographical version also generalizes models appearing in a variety of fields, such as the unit disk graph, the Boolean model, and the gravity model, as discussed in the presentation.

References:

- [1] Naoki Masuda, Hiroyoshi Miwa and Norio Konno, "Geographical threshold graphs with small-world and scale-free properties", submitted (2004). Preprint: cond-mat/0409378.
- [2] Naoki Masuda, Hiroyoshi Miwa and Norio Konno, "Analysis of scale-free networks based on a threshold graph with intrinsic vertex weights", Physical Review E, 70,

Preferential attachment growth model and nonextensive statistical mechanics

Luciano da Silva

Universidade Federal do Rio Grande do Norte, Brazil

We introduce a two-dimensional growth model where every new site is located, at a distance r from the barycenter of the pre-existing graph, according to the probability law $1/r_i^{\alpha_G}$ ($\alpha_G \geq 0$) and is attached to (only) one pre-existing site with a probability $\propto k_i/r_i^{\alpha_A}$ ($\alpha_A \geq 0$; k_i is the number of links of the i^{th} site of the pre-existing graph, and r_i its distance to the new site). Then we numerically determine that the probability distribution for a site to have k links is asymptotically given, for all values of α_G , by $P(k) \propto e_q^{-k/\kappa}$, where $e_q^x \equiv [1+(1-q)x]^{1/(1-q)}$ is the function naturally emerging within nonextensive statistical mechanics. The $\alpha_A=0$ particular case belongs to same universality class to which the Barabasi-Albert model belongs.

Directed network as a chaotic piece-wise linear one-dimensional map and its large-deviation properties

Syuji Miyazaki

Graduate School of Informatics, Kyoto University, Kyoto 606-8501, Japan

Directed network such as WWW can be represented by a stochastic matrix. Comparing this matrix to a Frobenius-Perron matrix of a chaotic piece-wise linear one-dimensional map whose domain can be divided into Markov sub-intervals, we are able to regard network structure itself as chaotic dynamics. Just like various large-deviation properties of local expansion rates (finite-time Lyapunov exponents) related to chaotic dynamics, we can also discuss those properties of network structure.

Complex network of seismicity

Norikazu Suzuki

*College of Science and Technology, Nihon University, 7-24-1 Narashinodai, Funabashi,
Chiba 274-8501, Japan*

The districts of southern California and Japan are divided into small cubic cells, each of which is regarded as a vertex of a graph if earthquakes occur therein. Two successive earthquakes define an edge and a loop, which replace the event-event correlation. In this way, the seismic data is mapped to evolving undirected and directed networks. It is shown that the connectivity distribution decays as a power law. Regarding the undirected network, the characteristic path length between two earthquakes chosen at random takes small value between 2 and 3. The clustering coefficient is found to be about 10 times larger than that in the case of the completely random network. For the directed network, two topologically-invariant statistical quantities are studied. One is the period distribution and the other is the path-length distribution. It is found that both of the distributions are scale free. These features highlight a novel aspect of seismicity as a complex phenomenon.

Complexity and nonextensivity in Hamiltonian long-range models

Andrea Rapisarda

*Dipartimento di Fisica e Astronomia, INFN sezione di Catania, Universita di Catania,
Via S. Sofia 64, 95123 Catania, Italy*

I will review some recent results on the anomalous dynamics of Hamiltonian long-range models. Connections to glassy systems and nonextensive thermodynamics will be also discussed.

Quasi-equilibrium evolution in self-gravitating N-body systems

Atsushi Taruya

*Research Center for the Early Universe (RESCEU), School of Science,
University of Tokyo, Hongo 7-3-1, Bunkyo-ku, Tokyo 113-0033, Japan*

In this talk, a quantitative characterization for the evolutionary sequence of stellar self-gravitating N-body system is discussed, focusing on the pre-collapse stage of the long-term dynamical evolution. In particular, we consider the quasi-equilibrium behaviors of the N-body systems in the setup of the so-called Antonov problem and try to seek a possible connection with non-extensive thermostatistics of self-gravitating systems. For this purpose, a series of long-term N-body simulations with various initial conditions are performed. We found that a quasi-equilibrium sequence away from the thermal equilibrium can be characterized by the one-parameter family of the stellar models. Especially, the stellar polytropic distribution satisfying the effective equation of

state $\propto \rho^{1+1/n}$, which is the extremum state of the Tsallis-type non-extensive entropy, provides a good approximation to the evolutionary sequence of the N-body system. Based on the numerical results, the physical reason and the timescales of quasi-equilibrium behavior are discussed from a thermostatistical point-of-view.

**Democratic temperature distribution and Local virial relation:
Two hypotheses for self-gravitating systems**

Yasuhide Sota

*Department of Physics, Ochanomizu University, Faculty of Science,
2-1-1 Ohtuka, Bunkyo-ku, Tokyo, Japan*

We propose two hypotheses which characterize the collisionless quasi-equilibrium state that realizes after the cold collapse of self-gravitating systems. The first hypothesis is the linear temperature-mass (TM) relation, which yields a characteristic non-Gaussian velocity distribution. The second hypothesis is the local virial (LV) condition, which, combining the linear TM relation, determines the unique mass density profile as $\rho(r) = \rho_0 r^{-4} e^{-r_0/r}$. This density profile is consistent with the data of cold collapse simulations and also with the data of observed surface brightness of a typical elliptical galaxy in the outer region. Two families of spherical and isotropic models, polytropes and King models, are examined from a viewpoint of these two hypotheses. We found that the LV relation imposes a strong constraint on these models. We also comment on the effect of the anisotropy of the velocity dispersion on our hypotheses.

Dynamical approach to superstatistics

Christian Beck

School of Mathematical Sciences, Queen Mary, University of London,

Mile End Road, London E1 4NS, UK

The behavior of various classes of complex systems can often be effectively described by a 'superstatistics', a superposition of ordinary Boltzmann factors with a fluctuating inverse temperature due to spatio-temporal variations of the environment. We present some simple dynamical models for superstatistical systems, and analyse their behaviour in terms of marginal stationary densities and correlation functions. We test some of our hypothesis using experimentally measured velocity time series in turbulent flows.

The dynamics of financial markets: A multi-timescale non-Gaussian model

Lisa Borland

Evnine-Vaughan Associates, Inc., 456 Montgomery Street, Suite 800, San Francisco,

CA 94104, USA

Financial markets exhibit certain statistical properties, such as power-law distributions of returns, volatility clustering, a kurtosis which decays slowly across time-scales, and a close to log-normal distribution of the instantaneous volatility. In this talk we propose a model for stock movements that is based on a type of statistical feedback across many time scales, resulting in a stationary non-Gaussian process which captures all of the features listed above, while employing just one source of Brownian

noise. The single-time version of this model allows closed form solutions for option pricing, and has been explored in detail for that purpose, yielding parsimonious results which fit well to empirical data.

A mechanism leading from speculative bubbles to crashes

Taisei Kaizoji

*Division of Social Sciences, International Christian University,
3-10-2 Osawa, Mitaka, Tokyo, 181-8585 Japan*

In this talk, I will present a mechanism of crashes of bubbles in the Japanese asset (stock and land) markets from econophysics point of view. We investigate quantitatively statistical properties of an ensemble of land prices and stock prices in Japan in the period corresponding to a period of bubbles and crashes. We found that the tail of the distributions of the ensembles of land prices and of stock prices in the high price range is well described by a power-law distribution, and furthermore, that as the power-law exponents approached unity, the bubbles collapsed.

Similarity and Probability Distribution Function in Stochastic Processes with Multiplicative Interactions

Akihiro Fujihara

*Graduate School of Integrated Science, Yokohama City University,
22-2 Seto, Kanazawa-ku, Yokohama 236-0027, Japan*

We have considered stochastic processes with multiplicative interactions analytically and numerically. The model is N-particle system and each particle has a positive quantity $x_i > 0$ ($i=1\dots N$). Randomly chosen two particles interact to transform their quantities from x_i and x_j to x'_i and x'_j ($i \neq j$) by a interaction rule. In this presentation, we mainly report two types of rules (1) asymmetric interaction : $x'_i = c(1-a)x_i + cbx_j$, $x'_j = da x_i + d(1-b)x_j$, where $c, d \geq 0$ and $0 \leq a, b \leq 1$ are interaction parameters. (2) random interaction : $x'_i = px_i + qx_j$, $x'_j = qx_i + px_j$, where p, q are random parameters with a probability distribution $\rho(p, q)$.

In numerical simulations, it is found that both of the models have similarity solutions scaled by the growth rate γ of the first moment of probability distribution function (PDF) and appear power-law tails in PDF in a certain region of the diagram of interaction parameters. Note that the power-law exponent s varies continuously depending on interaction parameters. We have determined the values of γ and s analytically by transcendental equations which is obtained by the master equation.

At the last, we will mention applications of our models to the theory of wealth distributions in econophysics.

**Non-universal finite size effects with universal infinite-size free energy
for the α -XY model**

GOTO Shin-itiro

Department of Applied Mathematics and Physics, Kyoto University, Japan

We study finite size effects in a family of systems in which a parameter controls interaction-range. In the long-range regime where the infinite-size free energy is universal, we show that the finite size effects are not universal but depend on the

interaction-range. The finite size effects are observed through discrepancies between time-averages of macroscopic variables in Hamiltonian dynamics and canonical averages of ones with infinite degrees of freedom. For a low energy regime, it is numerically shown that convergences become slower as the interaction-range becomes shorter. For a high energy regime, the relation to a pair of the discrepancies is theoretically predicted and numerically confirmed.

Changing Opinions in a Changing World: a New Perspective in Sociophysics

Alessandro Pluchino

*Dipartimento di Fisica e Astronomia, INFN sezione di Catania, Universita di Catania,
Via S. Sofia 64, 95123 Catania, Italy*

We propose a new model of opinion formation, the Opinion Changing Rate (OCR) model. Instead of investigating the conditions that allow consensus in a world of agents with different opinions, we study under which conditions a group of agents with a different natural tendency (rate) to change opinion can find agreement. The OCR is a modified version of the Kuramoto model, one of the simplest models for synchronization in biological systems, here adapted to a social context. By means of several numerical simulations we illustrate the richness of the OCR model dynamics and its social implications.

The relevance of memory in minority game

Wai-Chung Man

*Department of Physics, the University of Hong Kong, Pokfulam Road, Central,
Hong Kong*

The Minority Game (MG) has become a famous model in econophysics since it was introduced by Challet and Zhang in 1997; fruitful and exciting results have been revealed from this game and many of its variants. Memory, which is the historical records of past outcomes of the game and is the only public information agents rely to make their choices, has long been regarded to be relevant in the asymmetric phase of the game, but irrelevant in the symmetric phase. Here, we show by a large scale simulation that the memory is in fact also relevant in the symmetric phase. Removal of memory eliminates the periodic dynamics behavior of the game in this region and alters its properties. Our discovery shows that memory in many complex adaptive models are important and cannot be simply dumped away.

**Earthquake generation model exhibiting self-organized criticality:
Relation between fault surface structure and statistical properties**

Tomohiro Hasumi

*Department of Physics, Waseda University, 3-4-1 Ohkubo Shinjyuku-ku,
Tokyo 169-8555, Japan*

Earthquakes (EQs) are natural phenomena involving frictional slip and fracture processes on the fault surfaces. One of the established statistical properties is Gutenberg-Richter (GR) law that shows the power law. The slope of this distribution, called b-value, is nearly equal to 1. Burridge and Knopoff [1967] proposed a primitive spring-block EQ model called BK model. Carlson et al [1991] and many later works

showed the BK EQ model, behaving like self-organized criticality (SOC), yield GR law. However, these works largely have not considered the fault surface structure yet, and the b-values are still far from the observation value 1.

In the present paper, we numerically investigate a one-dimensional BK model with inhomogeneous fault surfaces consisting of self-affine fractals.

It is demonstrated that b-value is closer to observation result, increasing the Hurst (roughness) exponent H which is a representative value of self-affine fractals. In the presentation, we also discuss other statistical properties.

Biological system as a universal class of steady growth system

Kunihiko Kaneko

University of Tokyo, Komaba, Meguro, Tokyo 153-8902, Japan

We intend to understand life as a 'complex system', by unveiling universal features underlying all biological systems. For this purpose, we take a constructive approach, by setting up a simple system both experimentally and theoretically, and answer general questions on a biological system. After surveying this standpoint, I discuss some of recent studies along this line, both in theory and experiment.

First, I discuss universal statistical laws of chemical abundances in a cell that sustains recursive production. From theoretical studies of simple protocell models, discovered are a power law in average gene expression and log-normal distribution of the abundances of each chemical. Experimental verification of these laws is also presented.

Second, to discuss relevance of this phenotypic fluctuations to evolution, fluctuation-dissipation theorem in physics is generalized, to obtain relationship between phenotypic fluctuations and genetic evolution. The proposed relationship is confirmed both in

experiments and in model simulations. Genetic assimilation is also revisited from this viewpoint.

Third, adaptation in a cell is discussed as a universal property of a system that has potentiality in growth. Plasticity in cellular states is quantitatively discussed.

References, papers and books listed in <http://chaos.c.u-tokyo.ac.jp>

Generalized entropies and the physics of classical and quantum information

Angel R. Plastino

Physics Department, University of Pretoria, Pretoria 0002, South Africa

The physics of Information has been the focus of an increasing research activity in recent years. Generalized entropic measures provide useful mathematical tools in these lines of inquire. In the present contribution we apply the generalized q -entropies: (1) to investigate the correlations existing between the q -entropies (both total and relative) of quantum states (pure and mixed) of composite systems, on the one hand, and the amount of entanglement exhibited by those states, on the other one, and (2) to establish certain limitations on the type of information-related processes that can be performed by classical dynamical systems, by considering the behavior of generalized Kullback measures evaluated on pairs of solutions of the associated Liouville equation. This allow us to formulate classical analogues, at the level of Liouville dynamics, of quantum impossible processes, such as universal quantum cloning.

Deviation from local equilibrium distribution in one-dimensional lattice

dynamical systems

Akira Ueda

Osaka Pref. Univ., Osaka, Sakai, Japan

One-dimensional lattice dynamical systems are studied. When both the ends are attached with heat baths at different temperatures, heat conduction occurs and nonequilibrium steady states are eventually realized. We focus on deviation from local equilibrium shown by the stationary states. For a number of models and various types of heat baths, local momentum distributions are computed numerically, and magnitudes of the deviation are measured by cumulants and relative entropy. As a result, we find differences in asymptotic behavior in large systems. We also discuss some symmetry breaking properties.

Poster Presentations

Fundamental theorems on nonextensive entropies

Shigeru Furuichi

*Tokyo University of Science in Yamaguchi, 1-1-1 Daigakudori, Onoda City,
Yamaguchi 756-0884, Japan*

The uniqueness theorem for Tsallis entropy was shown in "H. Suyari, IEEE Trans. Inform. Theory, Vol.50, pp.1783-1787 (2004)" by introducing the generalized Shannon-Khinchin's axiom. In my presentation, this result is generalized and simplified in the following manners.

- (1) Generalization: The uniqueness theorem for Tsallis relative entropy is shown by means of the generalized Hobson's axiom.
- (2) Simplification: The uniqueness theorem for the Tsallis entropy is shown by simpler axiom than the above one.

After the study on the uniqueness theorem on nonextensive entropy and nonextensive relative entropy, we try to discuss information theory based on nonextensive entropies. To do so, we firstly introduce the nonextensive conditional entropy and then study on its properties such as chain rule and subadditivity. Historically, a chain rule and a subadditivity for the information function of type Y^β , which is one of the nonextensive (nonadditive) entropies, were derived by Z.Daróczy. In this presentation, introducing a different but natural conditional entropy based on Tsallis entropy, we firstly show the chain rules for the Tsallis type entropies which are also nonextensive

(nonadditive) entropies, as including the famous pseudoadditivity in nonextensive statistical physics as a special case. They give important and natural relations between Tsallis conditional entropy and Tsallis joint entropy. We secondly show the subadditivity of the Tsallis type entropies for the correlated two random variables in the case of $q \geq 1$ with the help of the generalized Shannon inequality. This idea enables us to further show the strong subadditivity of the Tsallis type entropies and then we can discuss on the entropy rate for Tsallis entropies. Since the chain rules and the subadditivities imply that the conditioning reduces Tsallis entropies, we are naturally able to define Tsallis mutual entropy and also Tsallis conditional mutual entropy. For these Tsallis mutual entropies, we again show the chain rules. Finally, so-called the data processing inequalities for Tsallis mutual entropies are shown.

Moreover, we define the parametrically extended quantum relative entropy based on nonextensive system, and show its fundamental properties such as monotonicity. We also define the relative operator entropy based on nonextensive system, and then we show the trace inequality between it and nonextensive quantum relative entropy which generalizes the Hiai-Petz inequality which plays an important role in quantum information theory.

The operator equality on the Tsallis relative operator entropy is also shown by considering the tensor product. This relation generalizes the pseudoadditivity for the Tsallis entropy. As a corollary of our operator equality derived from the tensor product manipulation, we show several operator inequalities including the superadditivity and the subadditivity for the Tsallis relative operator entropy. Our results are generalizations of the superadditivity and the subadditivity for the Tsallis entropy.

1/f fluctuations at the edge of Hamiltonian chaos

Tomoshige Miyaguchi

Waseda University, 3-4-1 Okubo Shinjuku-ku, Tokyo, Japan

A new dynamical system which is non-hyperbolic and area-preserving is introduced. Through numerical simulations, it is found that this system exhibits $1/f$ fluctuations. Moreover, calculating the eigenvalues of the Frobenius-Perron operator of the map analytically, it is found that this system has long time correlations and the theoretical prediction is in a good agreement with the numerical results.

Collective motion in a Hamiltonian dynamical system with mean field coupling

Hidetoshi Morita

*Department of Basic Science, Graduate School of Arts and Sciences,
University of Tokyo, 3-8-1 Komaba, Meguro-ku, Tokyo 153-8902, Japan*

Relaxation in the Hamiltonian dynamical system of mean field XY model is studied. We discover novel metastability that the macroscopic quantities (temperature and the amplitude of mean field) show periodic and quasi-periodic collective motion, even in the thermodynamic limit ($N \rightarrow \infty$). The periodic motion appears through Hopf bifurcation. We also obtain the phase diagram.

Scale-free network with geometrical structures

Satoru Morita

Shizuoka University, Johoku, Hamamatsu, Japan

The study of the structure of the underlying networks is very important to understand the functions of complex systems. Biological, social and computer networks share several features. A large number of networks have a power law degree distribution. This class of networks is called scale-free. Another property called small-world is seen in many networks. For a small-world network containing N vertices the average shortest-path length grows logarithmically, whereas for a usual d -dimensional lattice obeys the power-law form. In addition, many networks have clustered and hierarchical structures. This tendency is quantified by the local clustering coefficient. Here, we propose a simple method to generate scale-free networks embedded on d -dimensional hidden space. Moreover, in this model, each vertex has another hidden variable called "fitness". The link between the vertices is decided by their fitness and distance.

**The eigenvalue distribution of the time-evolution operator of
non-equilibrium systems**

Yuichi Nakamura

*Institute of Industrial Science, University of Tokyo, Komaba 4-6-1, Meguro,
Tokyo 153-8505, Japan*

We propose a new algorithm of analyzing the phase transition of a non-equilibrium system with a flow. The phase transition manifests itself in the eigenvalue distribution of the time-evolution operator. We discuss two deterministic one-dimensional cellular automata, the rule184 and the EBCA1 (the extended Burgers cellular automaton 1), as a discrete traffic-flow model. Under the periodic boundary condition, we can have the free phase and the congested phase.

We use a new time-evolution operator with the flow operator incorporated as a

perturbation. Specifically, we generalize the matrix elements corresponding to the flow of a specific car from unity to e^{g} . This technique was first introduced to an electron system to discuss the localization of electrons in a random potential, namely the Anderson localization, by making the transfer energy asymmetric [1]. We here apply it to classical systems.

Consequently, the eigenvalue distribution of the new time-evolution operator shows a remarkable difference between the free phase and the congested phase. We can thus regard the congested phase as a localized state and the free phase as an extended state. Furthermore, the distance of the eigenvalues from the origin is related to the average velocity and the flux.

[1] N. Hatano and D. R. Nelson, Phys. Rev. Lett. 77, 570 (1996); Phys. Rev. B 56, 8651 (1997).

**Anomalous diffusion of cosmic rays in magnetic field turbulence:
Linkage between diffusion statistics and turbulence statistics**

Fumiko Otsuka

Earth System Science and Technology, Kyushu University, Japan

Cosmic rays are highly energetic, electronically charged particles thought to be originated and accelerated in association with dynamic astrophysical phenomena. We study spatial diffusion of the cosmic rays by numerically time integrating equation of motion assuming presence of two dimensional compressional magnetic field turbulence. Based on the numerical results, we attempt to link statistical properties of the particle diffusion and that of the magnetic field turbulence.

One of the important parameters in our simulation is ratio of typical particle Larmor radius (r_L) to the field correlation length (L). When r_L/L is small, the particles

essentially gradient-B drift along equi-contour lines of the magnetic field strength, and thus the diffusion in this parameter regime can be well understood by analyzing statistics of magnetic field islands composed of these equi-contour lines. We numerically evaluate the statistics of the field islands such as probability density function of mean radius and fractal dimension of field islands, as functions of power-law index of the magnetic field turbulence.

If a particle is guided by a large scale island without making a complete rotation around the islands within observation time scale τ , its orbit will appear to be almost ballistic. On the other hand, a particle trapped by a small scale island appears as trapped if it makes many rotations around the island within τ . Thus both super-diffusion (for finite time scale τ) and sub-diffusion can take place. We find numerically and analytically the scaling law of the diffusion coefficient using the parameters obtained by analysis of the field islands statistics.

**Canonical partition function in anomalous systems described
by the κ -entropy**

Antonio M. Scarfone

*Dipartimento di Fisica, Politecnico di Torino, Corso Duca degli Abruzzi 24,
I-10129 Torino, Italy*

Through a formal manipulation of the distribution function obtained by applying the MaxEnt principle to the κ -entropy [G. Kaniadakis, Phys. Rev. E 66, 056125 (2002)], we derive the expression of the canonical partition function and discuss its main properties. It is shown that all the important macroscopic quantities associated with the system can be expressed employing only this quantity. The relationship with the free energy of the system is also discussed.

**The Boltzmann temperature and Lagrange multiplier in nonextensive
thermostatistics**

Tatsuaki Wada

*Department of Electrical and Electronic Engineering, Ibaraki University,
4-12-1 Nakanarusawa, Hitachi, Ibaraki, 316-8511 Japan*

The relation between the Boltzmann temperature and the Lagrange multiplier associated with energy average in nonextensive thermostatistics are considered. In Tsallis canonical ensemble, the Boltzmann temperature is dependent on energy through the probability distribution unless $q=1$. It is shown that the so-called 'physical temperature' is nothing but the ensemble average of the Boltzmann temperature.

Finite-difference Lattice Boltzmann methods of binary fluids

Aiguo Xu

Department of Physics, Kyoto University, Japan

Lattice Boltzmann Method (LBM) has become a viable and promising numerical scheme for simulating fluid flows. There are several options to discretize the Boltzmann equation: (i) Standard LBM (SLBM); (ii) Finite-Difference LBM (FDLBM); (iii) Finite-Volume LBM; (iv) Finite-Element LBM; etc. These kinds of schemes are expected to be complementary in the LBM studies. For multicomponent fluids, (i) most existing

methods belong to the SLBM, and/or based on the single-fluid theory; (ii) nearly all the studies are focused on isothermal and nearly incompressible systems. In our study, a two-fluid kinetic model, first proposed by L. Sirovich, is clarified and extended. Based on this kinetic model, FDLBMs for binary Euler equations and Navier-Stokes equations are formulated.

We consider a binary mixture with two components, A and B . The based discrete velocity model (DVM) is described by two indexes, k and i , where k denotes the k th group of discrete velocities with the same size v_k , i indicates the direction of the discrete velocity. The basic ideas in the formulation procedure of the FDLBMs are as follows: (i) The Chapman-Enskog analysis shows what properties the discrete Maxwellian distribution function $f_{ki}^{\leftarrow(0\rightarrow)}$ should follow; (ii) Those requirements tell the lowest order of the flow velocity \mathbf{u} in the Taylor expansion of $f_{ki}^{\leftarrow(0\rightarrow)}$; (iii) The highest rank of tensors of the particle velocity \mathbf{v} in the requirements on the truncated $f_{ki}^{\leftarrow(0\rightarrow)}$ determines the needed isotropy of the DVM. The present approach works for binary neutral fluid mixtures. One possibility to introduce interfacial tension is to modify the pressure tensors, which is implemented by changing the force terms. For binary fluids with disparate-mass components, say $m^A \neq m^B$, only if the total masses and temperatures of the two species are not significantly different, Sirovich's kinetic theory works, so do the corresponding FDLBMs. When the masses and/or the temperatures of the two components are greatly different, the two-fluid kinetic theory should be modified. In those cases, the Navier-Stokes equations and the FDLBMs are not symmetric about the two components, but the formulation procedure is straightforward.

Thermodynamical and informational structure of superstatistics

Takuya Yamano

*Department of Physics, Ochanomizu University, 2-1-1 Otsuka, Bunkyo-ku,
Tokyo 112-8610, Japan*

We focus our attention to the structure of recently proposed notion of superstatistics by Beck and Cohen. By them, the connection to the Tsallis statistics has been formally given provided the fluctuating field takes a form of the chi-squared distribution under the Gibbsian local equilibrium. In spite of some applications especially for turbulence, the rigorous discussion for the concept of superposition of statistical factors are still missing. We fill this gap by investigating thermodynamical relations satisfied by superstatistics and by exploring how we can interpret it in terms of information theoretical point of view. Universal relations for systems having fluctuating inverse temperatures are derived.

Dynamics of two-sign point vortices in positive and negative temperature states

Yuichi Yatsuyanagi

*Graduate School of Human and Environmental Studies, Kyoto University,
Yoshida Nihonmatsu, Sakyo, Kyoto 606-8501, Japan*

Dynamics of two-sign point vortices in two-dimensional circular boundary is examined via numerical simulations using a special-purpose computer for molecular dynamics simulations, MDGRAPE-2 [1].

The conserved parameters confirmed in the simulation include energy E and the angular impulse I . For characterization of the closed system we employ microcanonical statistics to introduce a parameter (inverse temperature) $\beta = d \log W(E,I) / dE$. Here

$W(E,I)$ is the density of states of $2N$ point vortices as determined by a large-scale numerical sampling of 10^7 states. The observed distribution $W(E,I)$ has a single peak at $E=E_0$ and $I=0$, so that the temperature of the system is negative at energy with $E>E_0$ [2]. Numerical results also indicate that β is proportional to $-E$ in high energy states.

Dynamics of vortex systems is examined starting from various initial distributions characterized by different β and $I = 0$. The observation shows that distributions of the positive and the negative point vortices become broad and mixed when $\beta > 0$. Non-neutral vortex systems consisting of discrete patches of positive and negative vortices tend to be uniformly neutralized in the asymptotic state.

On the other hand, when $\beta < 0$, merger between patches of the same sign vortices occurs and the time-asymptotic distribution is characterized by a positive clump and a negative clump plus a broad distribution of neutralized vortices. The energy spectrum of the asymptotic state shows two peaks in high and low energy regions. It is revealed that the slope of the peak in high energy region and the upper limit of the energy in the spectrum increases almost linearly with $-\beta$. The appearance of two peaks indicates that a part of the vortices gains the energy to form the clumps while the others lose the energy to keep the total energy constant. Thus, the peaks in high and low energy regions correspond the vortices in the clumps and in the background, respectively. It suggests the common and essential role of the background vortices in supporting the condensation of two-sign vortices as well as in assisting the generation of symmetric configuration of nonneutral plasma clumps [3].

[1] Y. Yatsuyanagi, Y. Kiwamoto, H. Tomita, M.M. Sano, T. Yoshida and T. Ebisuzaki, "Dynamics of two-sign point vortices in positive and negative temperature states", to be published in Phys. Rev. Lett. (2005).

[2] L. Onsager, Nuovo Cimento Suppl., 6, 279 (1949).

[3] A. Sanpei, Y. Kiwamoto, K. Ito and Y. Soga, Phys. Rev. E 68, 016404 (2003).