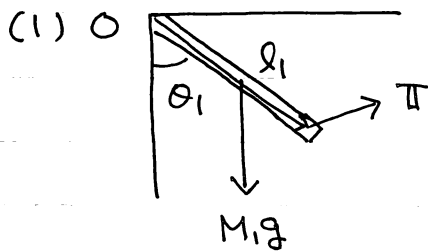
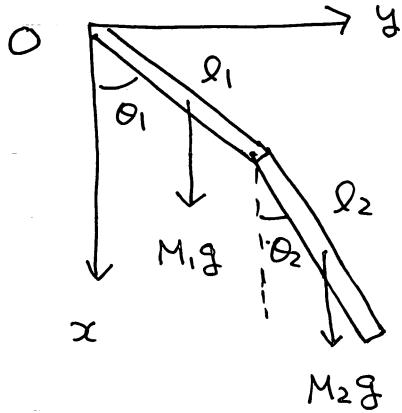


4-2 2重振り子



$$r = l_1 \cos \theta_1 \mathbf{e}_x + l_1 \sin \theta_1 \mathbf{e}_y$$

$$\mathbf{T} = T_x \mathbf{e}_x + T_y \mathbf{e}_y$$

Oまわりの \mathbf{T} の τ - \times ント

$$r \times \mathbf{T} = (l_1 \cos \theta_1 T_y - l_1 \sin \theta_1 T_x) \mathbf{e}_z$$

〃 重力の τ - \times ント

$$\frac{1}{2} r \times M_1 g \mathbf{e}_x = -\frac{1}{2} M_1 g l_1 \sin \theta_1 \mathbf{e}_z$$

従って $\frac{dL_z}{dt} = N_{iz}$, $L_z = I_1 \dot{\theta}_1$

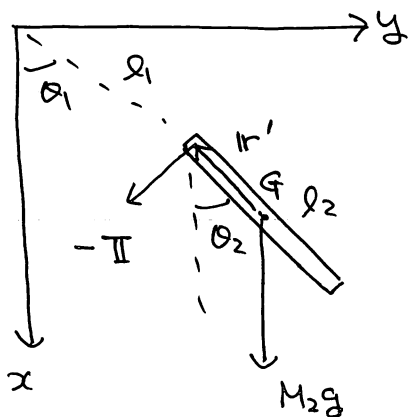
$$I_1 \ddot{\theta}_1 = l_1 \cos \theta_1 T_y - l_1 \sin \theta_1 T_x - \frac{1}{2} M_1 g l_1 \sin \theta_1$$

$$\uparrow = l_1 (T_y \cos \theta_1 - T_x \sin \theta_1) - \frac{1}{2} M_1 g l_1 \sin \theta_1 \quad (1)$$

棒1のOまわりの
慣性 τ - \times ント

$$I_1 = \frac{M_1}{3} l_1^2$$

(2)

棒2の重心'

$$x_{2G} = l_1 \cos \theta_1 + \frac{l_2}{2} \cos \theta_2$$

$$y_{2G} = l_1 \sin \theta_1 + \frac{l_2}{2} \sin \theta_2$$

$$\mathbf{II} = T_x \mathbf{e}_x + T_y \mathbf{e}_y$$

重心まわりの $-\mathbf{II}$ のモーメント

$$\mathbf{r}' = -\frac{l_2}{2} \cos \theta_2 \mathbf{e}_x - \frac{l_2}{2} \sin \theta_2 \mathbf{e}_y$$

$$\begin{aligned} \mathbf{r}' \times (-\mathbf{II}) &= \left(\frac{l_2}{2} \cos \theta_2 T_y - \frac{l_2}{2} \sin \theta_2 T_x \right) \mathbf{e}_z \\ &= \frac{l_2}{2} (T_y \cos \theta_2 - T_x \sin \theta_2) \mathbf{e}_z \end{aligned}$$

重心まわりの回転 $\frac{dL'_{zz}}{dt} = N'_{zz}$

$$I_{2G} \ddot{\theta}_2 = \frac{l_2}{2} (T_y \cos \theta_2 - T_x \sin \theta_2) \quad (2)$$

重心運動

$$M_2 \ddot{x}_{2G} = M_2 g - T_x$$

$$M_2 \ddot{y}_{2G} = -T_y$$

 \Rightarrow

$$T_x = M_2 g - M_2 \ddot{x}_{2G}$$

$$T_y = -M_2 \ddot{y}_{2G}$$

$$\begin{cases} x_{2g} = l_1 \cos \theta_1 + \frac{l_2}{2} \cos \theta_2 \\ y_{2g} = l_1 \sin \theta_1 + \frac{l_2}{2} \sin \theta_2 \end{cases}$$

$$\ddot{x}_{2g} = -l_1 (\ddot{\theta}_1 \sin \theta_1 + \dot{\theta}_1^2 \cos \theta_1) - \frac{l_2}{2} (\ddot{\theta}_2 \sin \theta_2 + \dot{\theta}_2^2 \cos \theta_2)$$

$$\ddot{y}_{2g} = l_1 (\ddot{\theta}_1 \cos \theta_1 - \dot{\theta}_1^2 \sin \theta_1) + \frac{l_2}{2} (\ddot{\theta}_2 \cos \theta_2 - \dot{\theta}_2^2 \sin \theta_2)$$

$$T_y \cos \theta_1 - T_x \sin \theta_1$$

$$= -M_2 g \sin \theta_1 + M_2 (\ddot{x}_{2g} \sin \theta_1 - \ddot{y}_{2g} \cos \theta_1)$$

$$= -M_2 g \sin \theta_1 - M_2 l_1 \ddot{\theta}_1 - \frac{M_2 l_2}{2} [\ddot{\theta}_2 \cos(\theta_1 - \theta_2) + \dot{\theta}_2^2 \sin(\theta_1 - \theta_2)] \quad (3)$$

$$T_y \cos \theta_2 - T_x \sin \theta_2$$

$$= -M_2 g \sin \theta_2 + M_2 (\ddot{x}_{2g} \sin \theta_2 - \ddot{y}_{2g} \cos \theta_2)$$

$$= -M_2 g \sin \theta_2 - \frac{M_2 l_2}{2} \ddot{\theta}_2 - M_2 l_1 [\ddot{\theta}_1 \cos(\theta_1 - \theta_2) - \dot{\theta}_1^2 \sin(\theta_1 - \theta_2)] \quad (4)$$

(1)(3) 5)

$$(I_1 + M_2 l_1^2) \ddot{\theta}_1 + \frac{1}{2} M_2 l_1 l_2 \cos(\theta_1 - \theta_2) \ddot{\theta}_2$$

$$= -\frac{1}{2} M_1 g l_1 \sin \theta_1 - M_2 g l_1 \sin \theta_1 - \frac{1}{2} M_2 l_1 l_2 \sin(\theta_1 - \theta_2) \dot{\theta}_2^2$$

(2)(4) 5)

$$I_2 \ddot{\theta}_2 + \frac{1}{2} M_2 l_1 l_2 \cos(\theta_1 - \theta_2) \ddot{\theta}_1$$

$$= -\frac{1}{2} M_2 g l_2 \sin \theta_2 + \frac{1}{2} M_2 l_1 l_2 \sin(\theta_1 - \theta_2) \dot{\theta}_1^2$$

$$I_2 = I_{2c} + \frac{1}{4} M_2 l_2^2$$