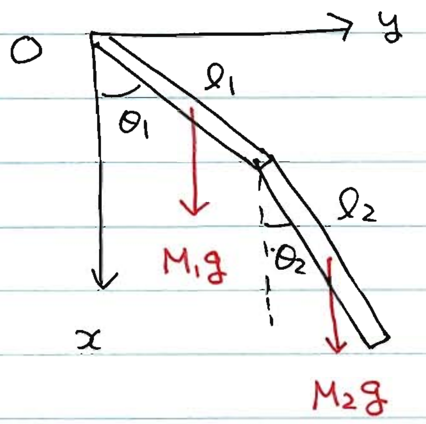
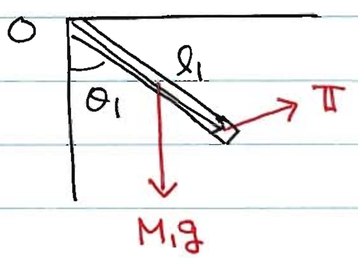
4-2　2重振り子



(1) 

$$\mathbb{r} = l_1 \cos\theta_1 \, \mathbb{e}_x + l_1 \sin\theta_1 \, \mathbb{e}_y$$

$$\mathbb{T} = T_x \, \mathbb{e}_x + T_y \, \mathbb{e}_y$$

Oまわりの $\mathbb{T}$ のモーメント

$$\mathbb{r} \times \mathbb{T} = (l_1 \cos\theta_1 \, T_y - l_1 \sin\theta_1 \, T_x) \, \mathbb{e}_z$$

〃　重力のモーメント

$$\frac{1}{2} \mathbb{r} \times M_1 g \, \mathbb{e}_x = -\frac{1}{2} M_1 g l_1 \sin\theta_1 \, \mathbb{e}_z$$

従って　$\dfrac{dL_{1z}}{dt} = N_{1z}$,　$L_{1z} = I_1 \dot{\theta}_1$

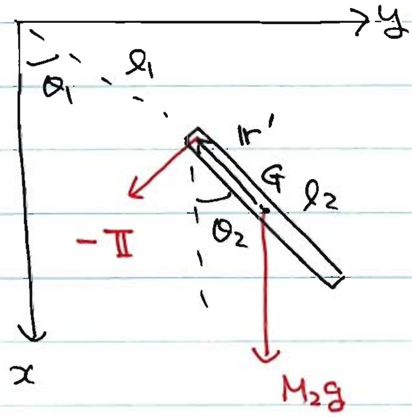
$$I_1 \ddot{\theta}_1 = l_1 \cos\theta_1 \, T_y - l_1 \sin\theta_1 \, T_x - \frac{1}{2} M_1 g l_1 \sin\theta_1$$

↑　$$= l_1 \underline{(T_y \cos\theta_1 - T_x \sin\theta_1)} - \frac{1}{2} M_1 g l_1 \sin\theta_1 \qquad (1)$$

棒1の O まわりの　$I_1 = \dfrac{M_1}{3} l_1^2$

慣性モーメント

(2)



棒2の重心

$$x_{2G} = l_1\cos\theta_1 + \frac{l_2}{2}\cos\theta_2$$

$$y_{2G} = l_1\sin\theta_1 + \frac{l_2}{2}\sin\theta_2$$

$$\mathbb{T} = T_x\,\mathbb{e}_x + T_y\,\mathbb{e}_y$$

重心まわりの $-\mathbb{T}$ のモーメント

$$\mathbb{r}' = -\frac{l_2}{2}\cos\theta_2\,\mathbb{e}_x - \frac{l_2}{2}\sin\theta_2\,\mathbb{e}_y$$

$$\mathbb{r}'\times(-\mathbb{T}) = \left(\frac{l_2}{2}\cos\theta_2\,T_y - \frac{l_2}{2}\sin\theta_2\,T_x\right)\mathbb{e}_z$$

$$= \frac{l_2}{2}\left(T_y\cos\theta_2 - T_x\sin\theta_2\right)\mathbb{e}_z$$

重心まわりの回転 $\quad \dfrac{dL'_{2z}}{dt} = N'_{2z}$

$$I_{2G}\ddot{\theta}_2 = \frac{l_2}{2}\left(\underline{T_y\cos\theta_2 - T_x\sin\theta_2}\right) \qquad (2)$$

重心運動

$$M_2\ddot{x}_{2G} = M_2 g - T_x$$

$$M_2\ddot{y}_{2G} = -T_y$$

$\Rightarrow$

$$\boxed{\begin{aligned} T_x &= M_2 g - M_2\ddot{x}_{2G} \\ T_y &= -M_2\ddot{y}_{2G} \end{aligned}}$$

$$
\left\{
\begin{aligned}
x_{2G} &= l_1 \cos\theta_1 + \frac{l_2}{2} \cos\theta_2 \\
y_{2G} &= l_1 \sin\theta_1 + \frac{l_2}{2} \sin\theta_2
\end{aligned}
\right.
$$

$$\ddot{x}_{2G} = -l_1 (\ddot{\theta}_1 \sin\theta_1 + \dot{\theta}_1^2 \cos\theta_1) - \frac{l_2}{2} (\ddot{\theta}_2 \sin\theta_2 + \dot{\theta}_2^2 \cos\theta_2)$$

$$\ddot{y}_{2G} = l_1 (\ddot{\theta}_1 \cos\theta_1 - \dot{\theta}_1^2 \sin\theta_1) + \frac{l_2}{2} (\ddot{\theta}_2 \cos\theta_2 - \dot{\theta}_2^2 \sin\theta_2)$$

$$T_y \cos\theta_1 - T_x \sin\theta_1$$

$$= -M_2 g \sin\theta_1 + M_2 (\ddot{x}_{2G} \sin\theta_1 - \ddot{y}_{2G} \cos\theta_1)$$

$$= -M_2 g \sin\theta_1 - M_2 l_1 \ddot{\theta}_1 - \frac{M_2 l_2}{2} \left[\ddot{\theta}_2 \cos(\theta_1 - \theta_2) + \dot{\theta}_2^2 \sin(\theta_1 - \theta_2) \right] \tag{3}$$

$$T_y \cos\theta_2 - T_x \sin\theta_2$$

$$= -M_2 g \sin\theta_2 + M_2 (\ddot{x}_{2G} \sin\theta_2 - \ddot{y}_{2G} \cos\theta_2)$$

$$= -M_2 g \sin\theta_2 - \frac{M_2 l_2}{2} \ddot{\theta}_2 - M_2 l_1 \left[\ddot{\theta}_1 \cos(\theta_1 - \theta_2) - \dot{\theta}_1^2 \sin(\theta_1 - \theta_2) \right] \tag{4}$$

(1)(3) より

$$(I_1 + M_2 l_1^2)\ddot{\theta}_1 + \frac{1}{2} M_2 l_1 l_2 \cos(\theta_1 - \theta_2)\ddot{\theta}_2$$

$$= -\frac{1}{2} M_1 g l_1 \sin\theta_1 - M_2 g l_1 \sin\theta_1 - \frac{1}{2} M_2 l_1 l_2 \sin(\theta_1 - \theta_2)\dot{\theta}_2^2$$

(2)(4) より

$$I_2 \ddot{\theta}_2 + \frac{1}{2} M_2 l_1 l_2 \cos(\theta_1 - \theta_2)\ddot{\theta}_1$$

$$= -\frac{1}{2} M_2 g l_2 \sin\theta_2 + \frac{1}{2} M_2 l_1 l_2 \sin(\theta_1 - \theta_2)\dot{\theta}_1^2$$

$$I_2 = I_{2G} + \frac{1}{4} M_2 l_2^2$$