Electrostatics in Wind-Blown Sand

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Introduction.—In wind-blown sand, or “saltation,” sand grains are propelled by the wind while they bounce along the surface (Fig. 1). Saltation plays a key role in various geological processes, including wind erosion, sediment transport, and the formation of sand dunes [1,2]. Moreover, the impact of saltating grains on the soil surface is the primary emission mechanism of mineral dust aerosols [2]. These small airborne dust particles affect the Earth system in several ways, including by scattering and absorbing radiation [3], acting as cloud seeds [4], and providing nutrients to ecosystems [5].

Pioneering measurements by Schmidt et al. [6] show that electric fields ($E$ fields) of up to $\sim 160 \text{kV/m}$ can be generated in saltation under moderate wind conditions. Laboratory experiments show that such $E$ fields facilitate the lifting of sand particles by winds, and can even directly lift sand particles from the surface [7]. Large $E$ fields are also predicted to occur in Martian saltation, possibly producing large quantities of hydrogen peroxide and making the Martian surface inhospitable to life as we know it [8].

It is known from industrial handling of powders that particle collisions tend to leave smaller particles with net negative charge and larger particles with net positive charge [9]. The physical mechanism governing this charge transfer is poorly understood, although various studies suggest that asymmetric rubbing (i.e., a small area of the small particle rubs over a large area of the large particle) causes a net transfer of electrons from the larger to the smaller particle [10]. We hypothesize that a similar process occurs in saltation, where particles bounce along a soil surface that can be interpreted as the surface of an infinitely large particle (Fig. 1). Saltating particles are thus expected to charge negatively with respect to the surface, as is indeed indicated by measurements of upward-pointing $E$ fields in wind tunnel and field experiments [6,11]. The occurrence of upward-pointing $E$ fields in dust devils and dust storms [12] also suggests negatively charged particles over a positively charged surface.

In this Letter, we present the first physically based numerical model of saltation that includes the generation of electric fields and the effects of electric forces on saltation. We show that recent measurements in saltation [13–15] cannot be explained by classical saltation theory [1,16], but are consistent with the predictions of our model when sand electrification is included.

Theory.—Saltation is initiated when the wind shear stress $\tau$ exceeds the threshold value $\tau_i$ necessary to move surface particles. The height-integrated particle mass flux $Q$ is a good measure of the intensity of saltation. Experiments show that $Q$ increases approximately cubically with wind shear velocity, $u^* = \sqrt{\tau / \rho_a}$ [1,2]; that is,

$$Q = Q_0 (\rho_a / g) u^{*3},$$

where $\rho_a$ is the air density, $g$ is the gravitational acceleration, and $Q_0$ is the dimensionless particle mass flux.

Model description.—A detailed description of our model is given in Ref. [17], but we describe it here briefly. We model saltation as the interplay of four processes [18]: (i) the motion of saltating particles, (ii) the modification of the wind profile through momentum transfer with saltating particles, (iii) the collisions of particles with the soil sur-

![FIG. 1. Schematic of saltation, showing the logarithmic wind profile $U(z)$ to the left of an idealized spherical sand particle propelled by the wind and bouncing along the surface. After lift-off from the surface, saltating particles obtain horizontal momentum from the wind, which is partially converted into vertical momentum after colliding with the surface and rebounding. The inset shows the force diagram of a negatively charged saltating particle over the positively charged soil surface.](image-url)
face, and (iv) the lifting of surface grains by wind stress and by particles impacting the soil surface. The main innovation of our model over previous models (e.g., [18]) is that the charging of saltating particles during collisions with other particles and the surface is included. The effect of the resulting electric forces on particle motion and the threshold shear stress ($\tau_\sigma$) is explicitly accounted for [7,17].

We model particle motion in two dimensions by considering gravitational, fluid, and electric forces (Fig. 1). The effects of turbulence and midair collisions on particle trajectories are neglected because these effects are relatively small for typical shear velocities [2,18]. Results of laboratory and numerical studies are used to model the collision of particles with the soil surface, including the ejection of surface grains [18].

For the calculation of the wind profile, we make the classical assumption that, in steady-state saltation, the fluid shear stress at the surface stays at the threshold value ($\tau_\sigma$) necessary to initiate the motion of surface particles [16,17]. The particle concentration per unit area is then given by [17]

$$N = \tau_p(0)/\tau_{sp}(0) = (\tau - \tau_{0})/\tau_{sp}(0),$$

where $\tau_p(0)$ is the total shear stress exerted by saltating particles at the surface, and $\tau_{sp}(0)$ is the average surface stress exerted by a single saltating particle, as computed from the particle trajectories. The size distribution of saltating particles is assumed to be similar to that of the parent soil for particles of 100–500 $\mu$m, in agreement with field measurements [17].

**Sand electrification.**—The model accounts for electrostatic charging of saltating particles during collisions with other particles [17] and the surface. Although collisional charge transfer between grains of granular material is observed in a variety of physical systems [6,9,11,12], the charging mechanism is not well understood. Nonetheless, Desch and Cuzzi [19] proposed a model in which the collisional charge transfer depends on the preexisting charges, the particle sizes, and the difference in the particles’ contact potential. They proposed that

$$q'_S = C_1(q_S + q_L) - C_2\Delta \Phi,$$

$$q'_L = (1 - C_1)(q_S + q_L) + C_2\Delta \Phi,$$

where $q_S$ and $q_L$ are the charges of the smaller and larger particles before the collision, $q'_S$ and $q'_L$ are the charges after the collision, $\Delta \Phi$ is the difference in particle contact potential, and $C_1$ and $C_2$ are functions of the mutual capacitances (and thus the radii) of the two particles, as defined by Eqs. 5–10 of Ref. [19]. For particles of similar composition (i.e., $\Delta \Phi = 0$), such as typical soil particles, (3) suggests that no charge transfer occurs when the colliding particles are not initially charged, which contradicts observations [6,9,11,12]. To mitigate this problem, we propose an effective contact potential difference between particle pairs of similar composition but different sizes. That is,

$$\Delta \Phi_{\text{eff}} = S(r_L - r_S)/(r_L + r_S),$$

where $S$ (in volts) is a physical parameter that scales the collisional charge transfer, and $r_S$ and $r_L$ are the radii of the small and large particles. This simple model has a functional form consistent with observations—smaller particles acquire net negative charge during collisions with larger particles, and the charge transfer is reduced as the relative difference in particle size decreases. Since saltating particles impacting the soil surface tend to interact with multiple surface grains [1,18], we interpret the soil surface as the surface of an infinitely large particle (i.e., $r_L = \infty$). By calibrating the model with $E$-field measurements in saltation [6], we found that $S = 6 \pm 4$ V (see Fig. 2).

The soil surface is assumed to be conducting, both because charge exchange with saltating particles provides charge mobility, and because conducting films of water are generally adsorbed on soils [7]. Since the height to which particles saltate is generally much smaller than the horizontal extent over which saltation occurs, we use the infinite plane approximation to determine the electric field $E$ from the calculated space charge density $\rho_e$ and soil surface charge density $\sigma$ [7].

$$E(z) = \frac{1}{2\varepsilon_0} \left( \sigma + \int_0^z \rho_e(z')dz' - \int_z^\infty \rho_e(z')dz' \right).$$

**FIG. 2.** Comparison of measured (squares) and modeled (solid line) $E$ fields in saltation as a function of height. The measurements [6] were taken for winds of 4–12 m/s at 1.5 m height, which was estimated to correspond to an average shear velocity of $0.5 \pm 0.1$ m/s [2]. The soil particle size distribution was taken as typical for the broad top of a dune [22], where the measurements were made. The inset shows the dependence of the surface $E$ field on shear velocity for the size distribution reported in Ref. [13], with the dashed line corresponding to the electric lifting threshold [7].
where \( \varepsilon_0 \) is the electric permittivity of air. The effect of the surface electric field on the threshold shear velocity (\( \frac{u^*}{0.0003} \)) is calculated using Eq. 9 in Ref. [7].

**Model results.** —Comparisons of vertical and horizontal profiles of the mass flux in saltation show that predictions of our model are in agreement with measurements (Fig. 3 and Ref. [17]). To the best of our knowledge, the model presented here is the first physically based model capable of accurately reproducing observed particle mass flux profiles. For high shear velocity, the agreement with field measurements improves when electric forces are included [Fig. 3(c)]. The height-integrated mass flux predicted by our model is also in good qualitative agreement with wind tunnel results [17].

Our model predicts that \( E \) fields increase sharply towards the surface and with wind speed (Fig. 2), in agreement with measurements [6,11]. The surface \( E \) field is of particular interest because \( E \) fields larger than \( \sim 80 \) kV/m reduce significantly the wind shear stress necessary to lift surface particles [7]. Our model finds that this effect contributes to an approximate doubling of the particle concentration due to sand electrification at large shear velocities (Fig. 4).

In addition to increasing the concentration of saltating particles, electric forces also affect particle trajectories. A characteristic height of saltation is the height \( z_{50} \) below which 50\% of the mass transport occurs. Classical saltation theory [1,16] predicts that increases in wind speed produce increases in the momentum of saltating particles, causing them to impact and rebound from the surface at higher speed, and therefore reach larger heights. However, recent measurements show that \( z_{50} \) stays approximately constant as the wind speed increases [13–15]. This clear discrepancy between measurements and theory can be resolved by the inclusion of sand electrification in our physically based saltation model (Fig. 5). As the negatively charged saltating particles bounce along the positively charged surface (Fig. 1) [6,11], the downward electric force causes particles to travel closer to the surface and at reduced horizontal speed [20]. Since the downward electric force increases with wind speed (Fig. 2), \( z_{50} \) remains approximately constant up to moderate shear velocities, in good agreement with measurements [13–15]. At larger shear velocities, electric forces become strong enough to lower the threshold shear velocity [7,17], which reduces the near-surface winds and thus \( z_{50} \). We plan to test this prediction with future field measurements.

**Conclusions.** —We developed the first physically based numerical saltation model that includes the effects of sand
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FIG. 5 (color). Dependence of the characteristic saltation height $z_{50}$ (see text) on the wind shear velocity. Classical saltation theory predicts that $z_{50}$ increases strongly with shear velocity [1,16], which our model also predicts when electric forces are not included (black circles). However, field measurements (squares [13] and triangles [14]) show that $z_{50}$ remains approximately constant. Inclusion of sand electrification in our model (red circles) resolves this discrepancy. The linear increase in $z_{50}$ without sand electrification is consistent with results from an independent numerical model [23]. Values of $z_{50}$ were obtained from Refs. [13,14] by fitting an exponential function to the measured vertical mass flux profiles, as described in Ref. [13]. Error bars represent the uncertainty in the fitting parameters. Model results were obtained for the size distribution reported in Ref. [13].

Although the current study focuses on terrestrial saltation, our model predicts that electric discharges occur during intense Martian saltation [24].

electrification. Significant discrepancies exist between classical saltation theory and field measurements [13]. We show that the inclusion of sand electrification in saltation models can resolve these discrepancies.

Model results show that sand electrification increases the particle concentration at a given wind shear velocity [7,20] (Fig. 4). Moreover, the downward electric force on saltating particles lowers their trajectories, improving the agreement between model predictions and measurements [13–15] (Fig. 5). Our results thus indicate that sand electrification plays an important role in saltation.

We are also investigating the effect of the classical assumption that the shear stress at the surface remains at the threshold value for particle entrainment [16,17]. Initial results suggest that removing this assumption could also explain some of the discrepancies between theory and measurements [21].

Although the current study focuses on terrestrial saltation, our model predicts that electric discharges occur during intense Martian saltation [24].

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[20] The decrease in particle speed due to electric forces reduces the average surface stress exerted by a single saltating particle $\tau_{e}(0)$ in (2), which contributes to the increase in particle concentration seen in Fig. 4.